A Motion-Based Communication System

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Abstract— **For some applications in team robotics, a wireless electronic communication system is not ideal. We propose for some of these tasks that it is more appropriate to communicate through motion, that is by encoding symbols in locomotion and decoding symbols using sensor data. We discuss some of the challenges and requirements of such a system and derive for the LTI case control policies used to enact trajectories that optimize a joint expression of control energy and robustness to observation noise.**

I. Introduction

Complex interactions between and emergent patterns of multiple agents can be achieved in the absence of any communication [1], [2]. Richer behavior, collaboration and more flexible control, however, can be achieved when direct communication between agents is allowed. For example, multiple agents tasked with efficiently finding targets in a given environment would clearly benefit from sharing with each other their best computed estimates of target positions. Typically such inter-agent communication is accomplished using some form of wireless electromagnetic transmission. Communication over wireless networks, both centralized and distributed, remains an active area of research and industrial application [3] and reasonably robust implementations of such communication systems are feasible for many robotic team applications [4], [5]. While wireless networks can offer high speed information transfer, as we discuss below, they are not always the best choice.

Wireless communication systems transmit information along frequency bands of the electromagnetic spectrum. They are therefore not suitable for underwater scenarios. Further, transmitted messages are subject to electromagnetic noise from the environment from natural sources such as solar phenomena [6]. Electromagnetic communication is also subject to cross-talk, in which signals from other sources bleed into the allotted communication channel. This problem is certain to become more frequent given the increasing prevalence of wireless systems [7]. Electromagnetic systems are also vulnerable to adversarial jamming [8] and other security risks [9]. It can be argued that any system operating in such environments should have a backup communication network that is robust to these error sources.

Perhaps more importantly, wireless communication systems require infrastructure. The physical mechanisms required to transmit/receive signals add both volume and payload to the design. Transmitting wireless signals requires consumption of energy, a resource that is not abundant for wireless systems, and the signal processing on received signals requires computational resources. These resource requirements are in direct conflict with the goal of developing teams of small, agile robotic agents that can operate in the field long-term [10], [11].

One method for inter-agent communication that does not require the transmission of wireless electromagnetic signals is motion-based communication (MBC). In such a paradigm, an agent that needs to transmit information will enact a trajectory. The transmitting agent's observing neighbors then measure its trajectory and decodes the desired message. This takes advantage of the agents' sensing and locomotion systems without requiring additional physical apparatus in order to communicate. A well-studied example of a MBC system in nature is the bee waggle dance [12] in which bees encode the direction and distance to a foraging target in the parameters of the dance.

While the fundamental study of MBC from a control point of view is still in its infancy, there is some prior work on communication through relative motion [13]–[15] and in communicating during shared activities such as dance [16]. This last also defines and analyzes an energy-minimal problem subject to particular separation constraints on the communicated symbols (see also Sec. II). Other works consider coupling information and continuous trajectories but do not endeavor to design cooperative communication systems. Results in symbolic control address the problem of finding minimum size alphabets of control primitives capable of steering several types of dynamical systems [17], but do not address control energy or observation. Others have studied decoding motions to determine a non-cooperative agent's intent [18] or to learn sequences of motion primitives [19], but have not consider the design of optimally decodable trajectories. There has also been work on the related problem of motion camouflage, that is finding trajectories that keep a system hidden from an observer [20].

In this work, we define and solve a minimum energy, maximum distinguishability problem for a linear system and connect it to the well-studied linear quadratic regulator (LQR) problem. The remainder of this paper is organized as follows. In the next section, we formulate and discuss the general MBC problem and define a specific optimal control problem that balances the amount of energy in the applied controls against the distinguishability of the encoded messages by the observer. In Sec. IV we restrict ourselves

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to linear time-invariant systems and solve the corresponding optimal control problem. We provide an example of our results in Sec. V. Finally, in Sec. VI we conclude with a discussion of the results and some future directions.

II. The MBC design problem

While there are many ways to formulate the MBC problem, there are several common issues and challenges including the following.

- Balance of tasks:
  Most cooperative tasks require agents to move in the mission space. If motion also encodes communication, there must be a way of making information-bearing motions distinct from informationless motions that are required to fulfill the task specification. One can consider decoupling task motions from communication motions either spatially (by reserving portions of the state space for each) or temporally. One can also consider superimposing a communication motion "on top" of a task motion by introducing, for example, a dither pattern on top of a translation.
- Scalability in number of agents:
  If an agent can observe a large number of neighboring agents, it must be able to receive messages from multiple neighbors at the same time and decode the signals; this is a strain on both sensor and computational resources. This is exacerbated by the fact that communicating agents must maintain line-of-sight (with respect to the sensors employed) and that agents may desire to simultaneously transmit and receive information.
- Limited energy consumption:
  Enacting a trajectory requires the consumption of energy by the agent. As discussed, in Sec. I, one motivation for MBC is its application on small, agile robotic agents in which energy must be carefully managed. The communication trajectories, then, should be as low-energy as possible.
- Scalability in message complexity:
  One of the most obvious restrictions of MBC is the limited bandwidth. One should not expect, then, to develop an MBC system which follows the same paradigm as for wireless communication through electromagnetic means. One appealing approach is to encode in the trajectories of an MBC system a limited number of possibly parameterized messages rather than a generic structure from which arbitrary messages can be built; such is the scheme in the bee waggle dance [12]. The particular messages will likely need to be carefully tailored for each situation.
- Balancing expressivity against distinguishability:
  Expressivity can be generated either by having a rich set of symbols, each conveying a different message, or by allowing complex messages to be generated by sequences of a small set of symbols. As discussed in the previous point, the limited bandwidth of MBC prevents the use of long sequences. The number of distinct symbols is also limited, in this case by the fact that the observer must be able to distinguish between them in face of sensor resolution and noise.

In this work we set aside the challenges of task balancing and scalability and focus on those of energy consumption and distinguishability. In the next section, we define a problem that captures these two issues and then focus it to the case of linear systems in Sec. IV.

III. MBC as an optimal control problem

Consider a simple model with a single transmitting agent T and a single observing agent O. T is a mobile agent whose dynamics are given by the general nonlinear differential equation

$$\dot{x} = f(x(t), u(t)), \quad x(t_o) = x_o, \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the (kinematic) state of T, $x_o \in \mathbb{R}^n$ is its initial state of T, and $u(t) \in \mathbb{R}^m$ is the control input to T. Let the kinematic state of O be given by a vector $z(t) \in \mathbb{R}^n$. O can make observations $y(t) \in \mathbb{R}^p$ of the state of T where the observation relationship is given by

$$y(t) = h(x(t), z(t)). \tag{2}$$

Note that for the purposes of this work we ignore the dynamics of the observing agent, though questions as to the best control for the observer to measure a transmission are both interesting and relevant.

Since we are ignoring issues related to combining communication with other tasks, in the sequel we will often suppress the dependence on the initial condition and fix the initial time to be $t_o = 0$. We also select a fixed communication interval $[0, t_f]$ and fixed final state x_f. One natural choice is to set the final state equal to the initial state; this would allow both for message concatenation and also ensure that the system can continue with whatever task it was performing prior to beginning a transmission. We naturally assume that x_f is reachable from the initial condition.

We suppose that we have a finite alphabet of communication symbols, $S = \{s_1, s_2, \ldots, s_q\}$. To each symbol we wish to associate a unique trajectory x_i and, by extension, an observed signal y_i. The MBC problem, then, is to select a set of control inputs $u_i(\cdot) : [0, t_f] \mapsto \mathbb{R}^m$ such that $x(t_f) = x_f$ and that minimize the energy utilized by T while maximizing the distinguishability (in a sense to be made precise in Sec. III-B below) of the signals observed by O.

A. Cost of transmission

We assume that associated to each symbol s_i is a probability p_i that the symbol will be selected for transmission. One component of our goal is to minimize the average value of the energy of transmission in the following sense. Let $J_i(t_f)$ be the total energy in the control signal $u_i(\cdot)$. Define the random variable $J(t_f)$ as the energy in the control signal of a randomly selected signal; J then takes on values in the set $\{J_i(t_f)\}_{i=1}^{q}$. The expected value of the total energy in sending a symbol is then given by

$$E[J(t_f)] = \sum_{i=1}^{q} p_i J_i(t_f). \tag{3}$$

Intuitively, a minimum energy assignment of controllers would associate the lowest energy control with the most probable symbol and, conversely the highest energy controller to the least probable symbol.

B. Distinguishability of symbols

We define the observation distance d_O between two signals $y_1(\cdot)$ and $y_2(\cdot)$ as

$$d_O(y_1,y_2;t_f)=\int_0^{t_f}(y_1(\sigma)-y_2(\sigma))^T M(y_1(\sigma)-y_2(\sigma))d\sigma \tag{4}$$

where ^{T} denotes transpose and M is a given symmetric, positive definite matrix. The "total distinguishability" of a set of symbols is denoted $\Delta(t_f)$ and defined by

$$\Delta(t_f)=\sum_{i,j:i\neq j} d_O(y_i,y_j;t_f). \tag{5}$$

Notice that contrary to the energy cost $E[J(t_f)]$, in the definition of $\Delta(t_f)$ we do not weight the distance between received signals by the frequencies of symbol selection. This is done to ensure that the communication system is uniformly robust.

Note also that we have chosen the L_2 norm to measure the distance between signals. While other norms can be selected, the choice of the L_2 norm provides a measure of robustness since measurement errors can be filtered out over the entire interval.

C. Optimal control problem

We can now bring together the cost functions of the control energy and the distinguishability to arrive at the optimal control problem capturing the MBC scenario.

$$\begin{aligned}&\min_{u_i}\ w_1E[J(t_f)]-w_2\Delta(t_f)\\ &\text{subject to}\\ &\qquad \dot{x}_i=f(x_i(t),u_i(t)),\\ &\qquad x_i(0)=x_o,\qquad x_i(t_f)=x_f,\\ &\qquad y_i(t)=h(x_i(t),z(t)),\\ &\qquad i=1,2,\dots,q.\end{aligned} \tag{6}$$

where w_1 and w_2 are weights that emphasize the importance of minimizing the control energy and of maximizing the distinguishability, respectively.

We note that the work in [16] sets up a similar problem but deals with the distinguishability objective by introducing a minimum distance constraint of the form

$$\min_{j\in\{1,2,\dots,q\}} d_O(y_i,y_j,t_f)\geq\delta_{min}>0\ \forall i\neq j \tag{7}$$

rather than using d_O in the objective to optimize the separation of the observed signals. The two approaches are clearly similar but we feel that the inclusion of the separation in the objective is more flexible. Note that with constraint (7), we expect the energy cost to increase monotonically with the parameter δ_{min} such that in the optimal case (7) will be satisfied with equality. In contrast, our formulation rewards increased distinguishability. Depending on the weights w_i, our approach may allow T to produce significantly more distinguishable signals by expending slightly more energy when compared to the approach in [16].

IV. MBC FOR LINEAR SYSTEMS

The general MBC problem defined in (6) is challenging to solve. In this section we simplify the problem to the case where T is a linear time-invariant (LTI) system and O has LTI observations, that is

$$\begin{aligned}\dot{x}(t)&=Ax(t)+Bu(t),\\ y(t)&=Cx(t).\end{aligned} \tag{8}$$

We assume that the pair (A,B) is controllable and that the pair (A,C) is observable. Applying the variation of constants of formula yields the solution

$$\begin{aligned}x(t)&=e^{At}x_0+\int_0^t e^{A(t-\sigma)}Bu(\sigma)d\sigma,\\ y(t)=Cx(t)&=Ce^{At}x_0+C\int_0^t e^{A(t-\sigma)}Bu(\sigma)d\sigma\end{aligned} \tag{9}$$

Using this in (4) with $M=I_n$ yields the observation distance

$$d_O(y_1,y_2,t_f)=\int_0^{t_f}(x_1(\sigma)-x_2(\sigma))^T C^TC\,(x_1(\sigma)-x_2(\sigma))\,d\sigma. \tag{10}$$

To complete the definition, we assume that the energy function associated with each controller is quadratic in the control and the state, that is

$$J_i(t_o,t_f)=\int_0^{t_f}u_i^T(\sigma)Ru_i(\sigma)+x_i^T(\sigma)Qx_i(\sigma)d\sigma \tag{11}$$

for given matrices $R=R^T>0$ and $Q=Q^T\geq 0$. The MBC problem (6) then becomes

$$\begin{aligned}&\min_{u_i}\int_0^{t_f}\Bigg(w_1\sum_{i=1}^q p_i\left(u_i^T(\sigma)Ru_i(\sigma)+x_i^T(\sigma)Qx_i(\sigma)\right)\\ &\qquad -w_2\sum_{i,j}(x_i(\sigma)-x_j(\sigma))^T C^TC\,(x_i(\sigma)-x_j(\sigma))\Bigg)d\sigma\\ &\text{subject to}\\ &\qquad \dot{x}_i(t)=Ax_i(t)+Bu_i(t),\\ &\qquad x_i(0)=x_o,\qquad x_i(t_f)=x_f,\\ &\qquad i=1,2,\dots,q\qquad j=1,2,\dots,q.\end{aligned} \tag{12}$$

This clearly has the form of the linear quadratic regulator (LQR) problem. To make this explicit, first define the stacked vectors $\tilde{u}(t),\tilde{x}(t),\tilde{x}_0$, and $\tilde{x}_f$ as

$$\begin{aligned}\tilde{u}(t)&=\begin{bmatrix}u_1^T(t) & u_2^T(t) & \cdots & u_q^T(t)\end{bmatrix}^T,\\ \tilde{x}(t)&=\begin{bmatrix}x_1^T(t) & x_2^T(t) & \cdots & x_q^T(t)\end{bmatrix}^T,\\ \tilde{x}_o&=\begin{bmatrix}x_o^T & x_o^T & \cdots & x_o^T\end{bmatrix}^T,\\ \tilde{x}_f&=\begin{bmatrix}x_f^T & x_f^T & \cdots & x_f^T\end{bmatrix}^T.\end{aligned} \tag{13}$$

Next define the combined state dynamics matrices $\tilde{A}$ (of dimension $nq \times nq$) and $\tilde{B}$ (of dimension $nq \times mq$) as

$$\tilde{A} = \text{diag}\{A, A, \cdots, A\}, \tag{14a}$$

$$\tilde{B} = \text{diag}\{B, B, \cdots, B\}. \tag{14b}$$

Then combine the observation matrix C with the observations weights to define $\tilde{C}$ (of dimension $nq \times nq$) as

$$\tilde{C} = \begin{bmatrix} 2w_2C^TC & -w_2C^TC & \cdots & -w_2C^TC \\ -w_2C^TC & 2w_2C^TC & \cdots & -w_2C^TC \\ \vdots & \ddots & \ddots & \vdots \\ -w_2C^TC & \cdots & -w_2C^TC & 2w_2C^TC \end{bmatrix}. \tag{15}$$

Finally, define the combined state and control cost matrices $\tilde{R}$ (of dimension $mq \times mq$) and $\tilde{Q}$ (of dimension $nq \times nq$) as

$$\tilde{R} = \begin{bmatrix} w_1p_1R & 0 & \cdots & 0 \\ 0 & w_1p_2R & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & w_1p_qR \end{bmatrix}, \tag{16a}$$

$$\tilde{Q} = \begin{bmatrix} w_1p_1Q & 0 & \cdots & 0 \\ 0 & w_1p_2Q & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & w_1p_qQ \end{bmatrix}. \tag{16b}$$

Putting all this together, we can rewrite the linear optimal control problem for MBC, (12) as

$$\begin{aligned} &\min_{u_i, i\in\{1,2,\dots,q\}} \int_0^{t_f} \left(\sum_{i=1}^{q} \left(\tilde{u}^T(\sigma)\tilde{R}\tilde{u}(\sigma) + \tilde{x}^T(\sigma)\left(\tilde{Q}-\tilde{C}\right)\tilde{x}(\sigma) \right) \right) \\ &\quad \text{subject to} \\ &\qquad \dot{\tilde{x}}(t) = \tilde{A}\tilde{x}(t) + \tilde{B}\tilde{u}(t), \\ &\qquad \tilde{x}(0) = \tilde{x}_o, \tilde{x}(t_f) = \tilde{x}_f, \\ &\qquad i = 1, 2, \dots, q. \end{aligned} \tag{17}$$

Problem (17) is the well-known continuous-time linear quadratic cost minimization problem with fixed endpoint. The solution to this problem is given by (see, e.g. [21])

$$\begin{aligned} &\tilde{u}(t) = -\tilde{B}^T K(t)\tilde{x}(t) - v(t), \\ &\dot{K}(t) = \tilde{A}K(t) + K(t)\tilde{A}^T + K(t)(\tilde{Q}-\tilde{C})K(t) - \tilde{B}\tilde{R}\tilde{B}^T, \\ &K(t_f) > 0, \\ &v(t) = -\tilde{B}^T\Phi_{\tilde{A}-\tilde{B}\tilde{B}^TK(t)}(0,t)\eta_0, \\ &W(\tilde{A}-\tilde{B}\tilde{B}^TK(t), \tilde{B}, 0, t_f)\eta_0 = \tilde{x}_o - \Phi_{\tilde{A}-\tilde{B}\tilde{B}^TK(t)}(0,t_f)\tilde{x}_f. \end{aligned} \tag{18}$$

where $\Phi_{\tilde{A}-\tilde{B}\tilde{B}^TK(t)}(0,t)$ is the state transition matrix associated with $\tilde{A}-\tilde{B}\tilde{B}^TK(t)$ and $W(\tilde{A}-\tilde{B}\tilde{B}^TK(t), \tilde{B}, 0, t_f)$ is the controllability Gramian associated with the pair $(\tilde{A}-\tilde{B}\tilde{B}^TK(t), \tilde{B})$. Since $\tilde{u}$ contains each of the control inputs defining each of the signaling trajectories, solving (18) yields the entire set of optimal controls $\{u_i^*\}_{i\in[1,q]}$ to encode the q messages.

In order for the solution in (18) to be optimal, the matrix $\left(\tilde{Q}-\tilde{C}\right)$ must be positive semidefinite. In the following proposition, we develop a sufficient condition for which positive semi-definiteness holds under the assumption that the probabilities of selection for each of the symbols are equal.

Proposition 1: Assume that the alphabet of q communication symbols has a uniform selection probability distribution, that is $p_i = \frac{1}{q}$, $i = 1, 2, \dots, q$. Then, if

$$\frac{w_1}{q}Q - 3w_2C^TC \geq 0, \tag{19}$$

then the matrix $\tilde{Q}-\tilde{C}$ is positive semidefinite.

Proof: For the given alphabet of q symbols, define the matrix ζ_q as

$$\zeta_q = \tilde{Q} - \tilde{C}$$

Then ζ_q can also be written as

$$\zeta_q = I_q \otimes \left(\frac{w_1}{q}Q - 3w_2C^TC\right) + 1_{q\times q} \otimes w_2C^TC \tag{20}$$

where $\otimes$ is the Kroenecker (tensor) product, I_q is an identity matrix of dimension $q \times q$ and $1_{q\times q}$ is an $q \times q$ matrix in which every element is 1. This decomposition of ζ_q yields the following relationship,

$$\begin{aligned} \tilde{x}^T\zeta_q\tilde{x} &= \sum_{i=1}^{q} x_i^T \left(\frac{w_1}{q}Q - 3w_2C^TC\right)x_i \\ &+ \left(\sum_{i=1}^{q} x_i\right)^T w_2C^TC \left(\sum_{i=1}^{q} x_i\right) \end{aligned} \tag{21}$$

Since C^TC is positive semidefinite, the second sum in (21) is guaranteed to be non-negative. By selecting w_1, w_2, and Q such that $\frac{w_1}{q}Q - 3w_2C^TC \geq 0$, the first sum is guaranteed to be non-negative. Under this condition, then, $(\tilde{Q}-\tilde{C}) \geq 0$ and the proposition is proved.

■

In general, one would not expect the communication symbols to be selected with uniform probability. The following extends Prop. 1 to the non-uniform case.

Proposition 2: Assume that the alphabet of q communication symbols has a given non-uniform selection probability distribution p_i, $i = 1, 2, \dots, q$. Define

$$p^* = \min_{i\in\{1,2,\dots,q\}} p_i.$$

Then, if

$$w_1p^*Q - 3w_2C^TC \geq 0,$$

then the matrix $\tilde{Q}-\tilde{C}$ is positive semidefinite.

Proof: Following the same procedure for Prop. 1 yields

$$\begin{aligned} \tilde{x}^T\zeta_q\tilde{x} &= \sum_{i=1}^{q} x_i^T \left(w_1p_iQ - 3w_2C^TC\right)x_i \\ &+ \left(\sum_{i=1}^{q} x_i\right)^T w_2C^TC \left(\sum_{i=1}^{q} x_i\right) \end{aligned} \tag{22}$$

and thus

$$\tilde{x}^T \zeta_q \tilde{x} \geq \sum_{i=1}^{q} x_i^T \left(w_1 p^* Q - 3 w_2 C^T C \right) x_i + \left(\sum_{i=1}^{q} x_i \right)^T w_2 C^T C \left(\sum_{i=1}^{q} x_i \right) \tag{23}$$

Thus, if $w_1 p^* Q - 3 w_2 C^T C \geq 0$, then $\tilde{x}^T \zeta_q \tilde{x} \geq 0$. Thus $\tilde{Q} - \tilde{C} \geq 0$. ■

Note that Prop. 1 is a special case of Prop. 2.

V. A Simulation Example

To illustrate these results, we created a program in MATLAB that can calculate $\tilde{u}$ for a given set of parameters. The differential equations in (17) and (18) were solved numerically using MATLAB's built-in ODE solver ode45 and MATLAB's boundary value problem solver bvp4c. We used the program to solve the following system over the interval $[0,1]$.

$$\dot{x}(t) = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix} x(t) + \begin{pmatrix} 0.5 & 0.1 \\ 0.05 & 1 \end{pmatrix} u(t),$$
$$y(t) = \begin{pmatrix} 1 & 0.5 \end{pmatrix} x(t).$$

We call the first coordinate of the state $x(t)$ position and the second coordinate velocity. The cost matrices for the state and controls were set to

$$Q = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}, \qquad R = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}.$$

The initial and final state values were set to

$$x_o = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \qquad x_f = \begin{pmatrix} 5 \\ 0 \end{pmatrix}.$$

For this example, we sought the controls for three communication symbols with selection probabilities of

$$p_1 = 0.5, \qquad p_2 = 0.3, \qquad p_3 = 0.2.$$

with optimization weights

$$w_1 = 9, \qquad w_2 = 1.$$

The optimal trajectories corresponding to the symbols in the trinary alphabet are shown along with their associated energy costs in Fig. 1. Note that the optimization automatically assigned the symbol with the lowest frequency (s_3) to a trajectory with the highest energy cost among the three determined by the algorithm.

The corresponding signals observed by O when T enacts the optimal trajectories are shown along with their L_2 separation in Fig. 2. Note here that the minimum pairwise L_2 separation occurs between the two least frequently selected signals y_2 and y_3 despite the fact that we did not explicitly consider selection frequency in the formulation of $\Delta(t_f)$. The total optimal cost is $w_1 E[J(t_f)] - w_2 \Delta(t_f) = 616.20$.

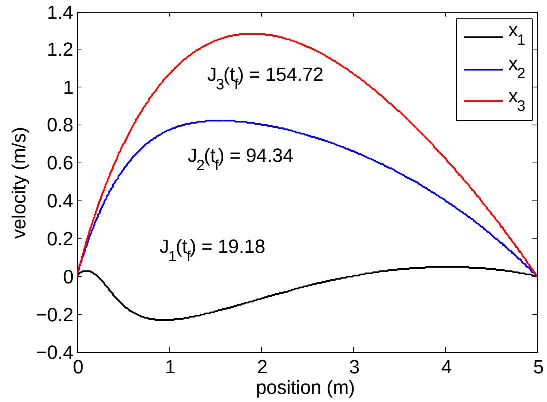

Fig. 1. Optimal trajectories in phase space found for a 3-symbol alphabet when (18) is applied to the system with given dynamics, cost and parameters (details are in the text). The energy cost of each motion is shown.. The expected energy is $E[J(t_f)] = 68.88$.

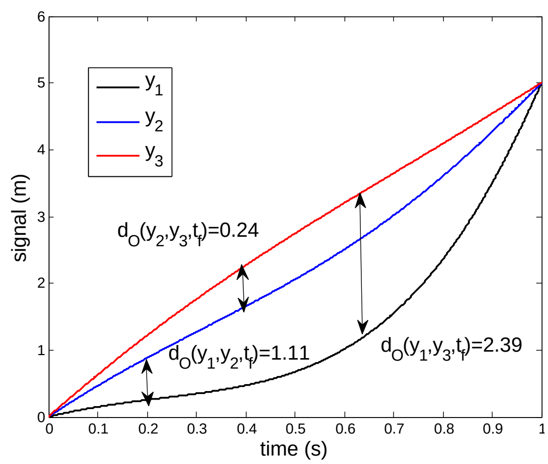

Fig. 2. The signals obtained from observing the optimal trajectories in Fig. 1 in the absence of noise. The values of the pairwise signal separation (in the L_2 norm) are noted on the graph. The total separation is $\Delta(t_f)$ =3.75

VI. Discussion

The propositions in Sec. IV give us sufficient conditions that enable the use of the LQR framework to find minimum energy, maximum separation trajectories for encoding signals in motion. This approach, however, does not necessarily scale well with alphabet size and cannot easily accommodate symbols whose use is infrequent. Consider, for example, condition (19) in Prop. 1. Given a fixed Q and C, as the number of symbols grows, $q \to \infty$, it must be that the ratio of the weights also tends to infinity, that is

$$\frac{w_1}{w_2} \to \infty.$$

From this we infer that the LQR approach is fundamentally limited in terms of the number of communication symbols it can support. As the number of symbols grows, the separation between those signals in the observation space diminishes, eventually resulting in indistinguishable symbols. One possible way around this is to replace the optimization over the signal separation with a constraint on the separation, as in [16], though intuitively one expects there to be a cost in energy to support the separation between all the symbols for very large alphabets.

Under the LQR approach, this result indicates that small alphabet sizes are preferred. This is an intuitively appealing notion as it is certainly easier to distinguish two symbols than 100. This implies that it may be interesting to consider alphabet size explicitly in the optimization problem. That is, as the weight of the energy used for the symbols (as captured by w_1) becomes large relative to the cost of separation (as captured by w_2), one may use a smaller alphabet and communicate messages through longer words. This decision would increase distinguishability but would likely increase the cost of sending entire messages. To capture this problem, define the function $L(q,m)$ to be the length of a message m encoded in an q-symbol alphabet. Then, if we know the selection probability of m we may instead change the objective of our minimization from the one in (12) to

$$\min_{q,\{u_i\}_{i\in\{1,2,\ldots,q\}}} \int_0^{t_f} \left(w_1 E_m \left[L(q,m) \sum_{i=1}^{q} u_i(\sigma)^T R u_i(\sigma) + x_i(\sigma)^T Q x_i(\sigma) \right] - w_2 \sum_{j,k:j\neq k} \left(x_j(\sigma) - x_k(\sigma)\right)^T C^T C \left(x_j(\sigma) - x_k(\sigma)\right) \right) d\sigma \tag{24}$$

where the expectation is now over the probabilities among *messages* rather than symbols.

Another issue arises when the selection distribution is non-uniform. For condition (2) in Prop. 2 to be met, the ratio of the weights must again go to infinity as $p^* \to 0$. Thus larger entropy distributions force a higher weight ratio and thus a stronger weighting on the energy at the expense of the separation. To circumvent this, one could initially ignore the selection frequency weights and redefine (3) to be

$$E[J(t_f)] = \sum_{i=1}^{q} \frac{1}{q} J_i(t_f) \tag{25}$$

so that the symbols are assumed to satisfy a uniform distribution. Since the distribution is not uniform, however, we would still like to optimize the symbol encoding to minimize the average amount of energy used in a signal transmission. This can be done by calculating the actual energy of each of the trajectories found and assigning the lowest energy trajectory to the most probable symbol, the next lowest energy trajectory to the second most probable, and so on.

VII. Conclusions

In this paper we introduced a version of the motion based communication problem in which both the cost of transmitting a symbol through a trajectory and the separation in the observation space between all the symbols in a given alphabet are optimized. We connected the problem to the LQR for the case of a linear system.

While these results are promising, the analysis also showed that there are concerns with this approach when one considers alphabets with large cardinality or a wide range in the selection frequencies of the symbols. While we proposed a few possible alternatives to overcome these issues, this remains an interesting and open topic.

References

[1] E. W. Justh and P. S. Krishnaprasad, "Equilibria and steering laws for planar formations," *Systems and Control Letters*, vol. 52, pp. 25–38, 2004.

[2] N. Moshtagh, N. Michael, A. Jadbabaie, and K. Daniilidis, "Vision-based, distributed control laws for motion coordination of nonholonomic robots," *IEEE Transactions on Robotics*, vol. 25, no. 4, pp. 851–860, 2009.

[3] C.-Y. Chong and S. Kumar, "Sensor networks: evolution, opportunities, and challenges," *Proceedings of the IEEE*, vol. 91, no. 8, pp. 1247–1256, August 2003.

[4] Z. Wang, M. Zhou, and N. Ansari, "Ad-hoc robot wireless communication," in *IEEE International Conference on Systems, Man and Cybernetics*, vol. 4, October 2003, pp. 4045–4050.

[5] A. Winfield and O. Holland, "The application of wireless local area network technology to the control of mobile robots," *Microprocessors and Microsystems*, vol. 23, no. 10, pp. 597–607, 2000.

[6] L. J. Lanzerotti, "Solar and solar radio effects on technologies," in *Solar and Space Weather Radiophysics*, ser. Astrophysics and Space Science Library, D. E. Gary and C. U. Keller, Eds. Springer Netherlands, 2005, vol. 314, pp. 1–16.

[7] H. Yang, "Analysis of rf radiation interference on wireless communication systems," *IEEE Antennas and Wireless Propagation Letters*, vol. 2, no. 1, pp. 126–129, 2003.

[8] W. Xu, W. Trappe, Y. Zhang, and T. Wood, "The feasibility of launching and detecting jamming attacks in wireless networks," in *Proceedings of the 6th ACM international symposium on Mobile ad hoc networking and computing*, ser. MobiHoc '05, 2005, pp. 46–57.

[9] W. Arbaugh, "Wireless security is different," *Computer*, vol. 36, no. 8, pp. 99–101, August 2003.

[10] E. Şahin, "Swarm robotics: From sources of inspiration to domains of application," in *Swarm Robotics*, ser. Lecture Notes in Computer Science, E. Şahin and W. Spears, Eds. Springer Berlin / Heidelberg, 2005, vol. 3342, pp. 10–20.

[11] R. Bogue, "Microrobots and nanorobots: a review of recent developments," *The Industrial Robot*, vol. 37, no. 4, pp. 341–346, 2010.

[12] A. M. Wenner, P. H. Wells, and F. J. Rohlf, "An analysis of the waggle dance and recruitment in honey bees," *Physiological Zoology*, vol. 40, no. 4, pp. 317–344, 1967.

[13] D. V. Raghunathan and J. Baillieul, "Exploiting information content in relative motion," in *Proceedings of the American Control Conference*, 2009, pp. 2166–2171.

[14] ——, "Motion based communication channels between mobile robots," in *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2009, pp. 702–708.

[15] ——, "Relative motion of robots as a means for signaling," in *Proceedings of the International Conference on Intelligent Automation and Robotics*, 2008, pp. 710–714.

[16] J. Baillieul and K. Özcimder, "The control theory of motion-based communication: problems in teaching robots to dance," in *Proceedings of the American Control Conference*, 2012, pp. 4319–4326.

[17] A. Bicchi, A. Marigo, and B. Piccoli, "Feedback encoding for efficient symbolic control of dynamical systems," *Automatic Control, IEEE Transactions on*, vol. 51, no. 6, pp. 987 – 1002, june 2006.

[18] A. Bicchi, A. Fagiolini, and L. Pallottino, "Towards a society of robots," *Robotics Automation Magazine, IEEE*, vol. 17, no. 4, pp. 26 –36, dec. 2010.

[19] S. Niekum, S. Osentoski, G. Konidaris, and A. Barto, "Learning and generalization of complex tasks from unstructured demonstrations," in *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems*, October 2012. [Online]. Available: http://lis.csail.mit.edu/pubs/niekum-iros12.pdf

[20] E. W. Justh and P. S. Krishnaprasad, "Steering laws for motion camouflage," *Proceedings of the Royal Society A*, vol. 462, pp. 3629–3643, 2006.

[21] R. W. Brockett, *Finite Dimensional Linear Systems*. John Wiley and Sons, 1970.